Janvier 2010 (corrigé sommaire)

Ex 1

1) Réponse indicielle du 1er ordre : K ? τ ?

$K = \frac{5}{1} = 5$; $\tau = t_{y=3,15} \simeq 2s$

donc EDO : $\begin{cases} 2y' + y = 5u \\ y(0) = 0. \end{cases}$

2) EH : $y_h(t) = Ce^{-2t}$

Sol part : $y_p(t) = y_\infty = 5$

$\Rightarrow y(t) = Ce^{-2t} + 5$; $y(0) = C + 5 = 0 \Rightarrow C = -5$

$\boxed{y(t) = (1 - e^{-2t})5}$!

Ex 2 **2.1**

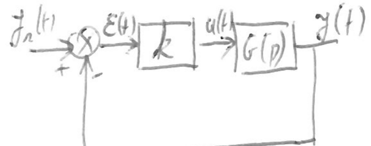
1) $y' = u \Rightarrow pY(p) - y(0) = U(p) \Rightarrow \boxed{\frac{Y(p)}{U(p)} = \frac{1}{p} = G(p)}$

2) $y' = 1 \Rightarrow y(t) = t + k$; $y(0) = 1 \Rightarrow k = 1$

$\Rightarrow \boxed{y(t) = t+1}$.

3) Non EBSB car $y(t)$ non bornée lorsque $u(t) = 1$!

2.2

1) 

2) $FTBF(p) = \frac{kG(p)}{1+kG(p)} = \boxed{\frac{k}{p+k}}$ $\left(= \frac{1}{1+\frac{1}{k}p} \right)$

3) $\varepsilon_p = 0 \ \forall k$ car $\frac{1}{p}$ dans $G(p)$! (visible aussi car $K_{BF} = FTBF(0) = 1$)

$\varepsilon_t = \lim_{p \to 0} \frac{u_0}{p}(1 - FTBF(p)) = \lim_{p\to 0} \frac{1}{p}\left(1 - \frac{k}{p+k}\right) u_0 = \lim_{p \to 0} \frac{1}{p+k} = \boxed{\frac{u_0}{k}}$

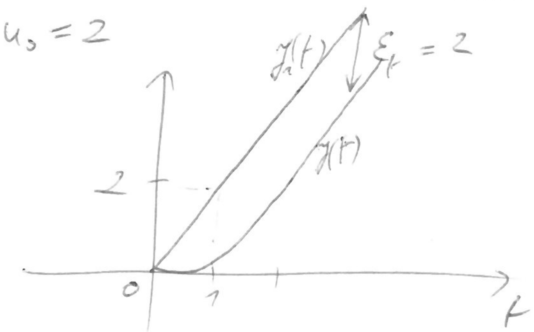
donc :

$\varepsilon_t \leq 0,05 u_0 \Leftrightarrow \boxed{k \geq \frac{1}{0,05} = 20}$

4) $t_r \leq 6 \Leftrightarrow \tau_{BF} \leq 2s$; or $\tau_{BF} = \frac{1}{k} \leq 2 \Rightarrow \boxed{k \geq \frac{1}{2}}$

5) $k = 1$ et $u_0 = 2$

donc $\varepsilon_t = 1 \times 2 = 2$



2.3

$\frac{U(p)}{\varepsilon(p)} = \frac{k^2}{p+2ak} \Leftrightarrow pU(p) + 2akU(p) = k^2\varepsilon(p)$

$\downarrow \mathcal{L}^{-1}$

$\boxed{\begin{cases} u'(t) + 2ak\,u(t) = k^2\varepsilon(t) \\ u(0) = 0 \end{cases}}$

2) $FTBF(p) = \frac{D(p)G(p)}{1 + D(p)G(p)} = \frac{k^2}{p(p+2ak) + k^2} = \frac{k^2}{p^2 + 2akp + k^2}$

3) $\omega_n = k$; $K_{BF} = 1$; $\xi_{BF} = a$

4) $\xi_{BF} = 1 \Leftrightarrow \boxed{a = 1}$

Ex 3

1) cf cours ! Contre-ex : $y^2 + y' = u$

2) cf cours !