Simple and Efficient Control of MEMS by Means of Operatorial Transformations

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***Abstract*—We present simple and original techniques well suited for designing efficient control strategies of any type, for a generic class of MEMS. These techniques are particularly well adapted when high performances and precision are required, because they are intensively based on the predictive capabilities of the model without needing its resolution, which make the so-obtained controls both simple to implement and robust.**

I. Introduction

This paper deals with the problem of control of MEMS governed by a dynamic equation (in general nonlinear) of the form:

$$\frac{d^2x(t)}{dt^2} = g\left(u(t), x(t), \frac{dx(t)}{dt}\right), \tag{1}$$

where u is the control, to be designed in such a way that (1) satisfies some desired properties.

Three kinds of control strategies can mainly be considered. In the first one, the function u is determined *a priori* ("open loop" control) on a time interval $[0, T]$, in aim of insuring, for example, that the evolution of the state $(x(t), \dot{x}(t))$ of (1) follows an ideal trajectory from initial conditions supposed to be known. Such an out-line determination of u can be achieved for example by minimizing a suitable cost functional, what in general necessitates to numerically solve the dynamic equation (1). This in general makes the minimization problem rather complex, with possibly a lot of local minima.

In the second one, u is determined in real time ("closed loop" control) under the form:

$$u = \mathbf{K}(x, x_r) \tag{2}$$

where $\mathbf{K}$ is a feedback *causal operator* and x_r is a reference trajectory, which can be chosen in real time or simply fixed as a constant. In such control strategies, a sensor is of course necessary to get informations about $x(t)$ (or even $\dot{x}(t)$) at any time (or at least at some discrete times t_n when digital computation is involved). For example, the well-known PI controllers are defined by:

$$u(t) = K_p\,(x(t) - x_r(t)) + K_i \int_0^t (x(s) - x_r(s))\,ds,$$

where the coefficients K_p, K_i must be chosen such that $x(t)$ satisfies some suitable dynamic properties. Note that when the model is highly nonlinear, such simple feedbacks can be difficult or even, sometimes, impossible to design.

Simple controls can also be constructed by use of operational amplifiers, the transfer function of which is ideally, in the linear range:

$$s = K(e^+ - e^-),$$

with $K \gg 1$, which naturally maintains the error signal $|e^+ - e^-|$ at a small level, but *only* if the closed-loop system can be insured to be *stable*. However, in this case also, nonlinear components in the system can often generate some ill-controlled dynamic behaviors resulting in instabilities. Furthermore, because they do not use the predictive properties of the system model, such simple feedbacks in general cannot insure both high precision and a sufficient insensitivity to perturbations, and so present poor dynamic performances.

In the last case, both strategies are used, that is the reference $x_r(t)$ is determined outline from minimization of a cost functional using the predictive properties of the model, and then it is implemented by means of a feedback control of the form (2) to correct some imperfections or perturbations not taken into account in the model (1). When it can be correctly implemented, such a strategy is of course the most efficient when high precision is required.

Various control methods for MEMS will be found for example in [2], [7], [10], [13], [14]. In any case, starting from a precise model (1), allowing good predictive properties about the evolution of the physical system is highly desirable, for example to make accurate simulations for designing the control strategy. Note that such accurate models can be beforehand elaborated from physical measurements by means of specific identification methods, see for example [6], [5].

In the sequel, we present simple and original methods well suited for designing efficient control strategies of any type, for a generic class of MEMS whose evolution is governed by (1). These methods are particularly well adapted when high performances and precision are required, because they are intensively based on the predictive capabilities of the model (1), but *without needing any resolution* of this differential equation. So, these methods remain both simple to implement and sophisticated and robust. They can be interesting alternatives to classical methods whose complexity is often an excessive shortcoming, particularly when stringent nonlinearities are involved with complex dynamic behaviors. A general statement of these "operatorial" approaches and some of their applications can be found in [11], [12].

The following generic model for electrostatically actuated MEMS is considered in the sequel:

$$\frac{d^2x}{dt^2} - g\left(x, \frac{dx}{dt}\right) - f(x) \cdot h(u) = 0. \quad (3)$$

Remark 1: In general, the control components u_i are positive because the MEMS is electrostatically actuated: electrostatic forces are proportional to the square of a voltage.

The main characteristic of such models lies in the fact that the electrostatic control $u \in \mathbb{R}^m$ is coupled with the state $x(t) \in \mathbb{R}^n$ via the function f (which can be highly non linear). This can generate serious difficulties for designing a robust control strategy when stringent performances are desired. The control problem under consideration can be of any nature: optimal control, trajectory planning, stabilization around a stable or unstable equilibrium point, etc. Functions g and f can involve nonlinear or even discontinuous components, in order to take into account various difficulties such as dry friction, hysteresis, etc., for which classical methods of nonlinear control are most of time ill adapted. For example, electrostatically actuated micro-mirrors such as described in [3], [4], [5], [6] can be described by (3) with suitable functions f and g.

II. Time-scale transformations for feedback design

In this section, we consider the particular case (frequently encountered in practice) where f is a positive scalar function and h is a $\mathbb{R}^n$-valued function.

By introducing $x_1 := x$, $x_2 = x$, model (3) is equivalently rewritten under the first order differential system:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = g(x_1, x_2) + f(x_1)\, h(u). \end{cases} \quad (4)$$

The basic idea of the method is to use suitable *operatorial* transformations of the equation (4) in order to get an equivalent model structurally adapted to the control problem. We designate by "operatorial transformation" any transformation acting on (4) considered as a *global* equation, that is relating to *trajectories* $X := (x_1, x_2)^T$ as functions defined on a predefined time interval $[0, T]$. Such transformations differ from classical transformations in the state-space, of the form $X(t) \mapsto G(X(t))$ with G a function in the classical sense; hence, we consider here operators $\mathbf{G}$, i.e. some correspondences acting on the whole trajectories:

$$X \mapsto \mathbf{G}(X).$$

An important non trivial example of such transformations, which will be used in the sequel, is the *Time Scale Transformation* (TST), defined from an increasing function $\varphi : [0, T] \to [0, \mathcal{T}]$ by the composition:

$$X \mapsto \tilde{X} = X \circ \varphi^{-1},$$

that is, and by defining the *new time* $\tau := \varphi(t)$, we have for any $\tau \in [0, \mathcal{T}]$:

$$\tilde{X}(\tau) := X(\varphi^{-1}(\tau)).$$

So, the trajectory X defined on the time interval $[0, T]$ is transformed into a new trajectory $\tilde{X}$ now defined on the interval $\tau \in [0, \mathcal{T}]$ and we have the one-to-one correspondences:

$$\tilde{X}(\tau) = X(\varphi^{-1}(\tau)), \quad X(t) = \tilde{X}(\varphi(t)). \quad (5)$$

From standard rules of differential calculus, we deduce the fundamental property:

$$\widetilde{X'} = \tilde{X}'\, \widetilde{\varphi'}. \quad (6)$$

Such time-scale transformations have been studied and applied to various problems in [11], [12].

Then, we introduce the TST defined by $\frac{d\varphi}{dt} = f(x_1)$; so, the new time τ can be easily computed (possibly in real time) by:

$$\tau = \varphi(t) = \int_0^t f(x_1(s))\, ds.$$

By noticing that $\widetilde{\varphi'} = f(\tilde{x}_1)$ and using (6), we get the new equivalent model (expressed in the "new" time τ):

$$\begin{cases} \dfrac{d\tilde{x}_1}{d\tau} = \dfrac{\tilde{x}_2}{f(\tilde{x}_1)} \\ \dfrac{d\tilde{x}_2}{d\tau} = \dfrac{g(\tilde{x}_1, \tilde{x}_2)}{f(\tilde{x}_1)} + h(\tilde{u}). \end{cases} \quad (7)$$

Thus, we have the remarkable property that the input $\tilde{u}$ and the state variables $\tilde{x}_i$ are now decoupled, which will significantly simplify the design of a (feedback) control.

III. Trajectorial parametrization of predictive control problems

We now consider generic models of the form:

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{x_2}{q(x_1)} \\ \dfrac{dx_2}{dt} = g(x) + f(x) \cdot h(u), \end{cases} \quad (8)$$

with q a positive scalar function[1].

Let us consider the (fictive) output:

$$y = x_1$$

(called parametric output [11]); this output has the remarkable property that if the following hypothesis are satisfied[2]:

$f(x(t))$ is an invertible matrix (for any $x(t)$),
h is an invertible function,

then the model (7) is entirely parametrized by y in the sense that both the state (x_1, x_2) and the input u are deduced from y without need of integrating the differential system. It can

[1] Possibly derived from a preliminary time-scale transformation as presented above.

[2] Such hypothesis are frequently satisfied in practice.

indeed be easily verified from simple formal computations that for any choice of the trajectory y (such that the above hypothesis is satisfied), the following trajectory (u, x_1, x_2) is the (unique) solution of (7):

$$\begin{cases} x_1 = y, \\ x_2 = q(y)\,\dfrac{dy}{dt}, \\ u = h^{-1}\left(f\,(x)^{-1} \cdot \left[\dfrac{dy}{dt}\,\nabla q(y) \cdot \dfrac{dy}{dt} + q(y)\,\dfrac{d^2 y}{dt^2} - g(x) \right] \right). \end{cases} \tag{9}$$

This last relations can be interpreted as an explicit model (equivalent to (8)) devoted to *control purposes* (while (8) can be seen as the model for simulation). They allow for example to construct open-loop predictive optimal controls by simply minimizing a suitable cost functional with respect to the trajectorial parameter y, without necessitating any integration of the differential model (7). It is to be noted that such controls are both precise (up to the accuracy of the model) and easily (and robustly) computed, even in cases where the function g is singular, for example when *dry friction* is involved, or even in the more general case where g is an hysteresis operator.

Remark 2: In the case where a time-scale transformation has been previously performed, the effective input is directly deduced using the inverse time-scale transformation.

In the sequel, we present and discuss some concrete examples of controls elaborated following the approach introduced above.

IV. Examples of predictive controls of MEMS

In this section, we briefly present some examples of possible controls based on the notions introduced above. These controls are elaborated from a realistic model of electrostatically actuated MEMS described here-after.

A. The MEMS under consideration

The system under consideration is an electrostatically actuated micromirror, a view of which[3] is given in Fig. 1. It is composed in two assembled parts. The upper one is a thin plate, the mirror, linked to a thick external rigid frame by two thin and narrow arms, the springs. This part is tailored in the same microcrystalline silicon layer of a SOI (Silicon On Insulator) wafer. The lower one comprises a balance-knife-edge with two electrodes distributed on both sides of it. The two parts are assembled in such manner that the axis of the springs and balance-knife-edge are identical. So the electrodes are located underneath the mirror inducing its rotation (left or right) when a voltage V is applied. The physical limit angle the mirror can reach is denoted α, whereas $\theta(t)$ denotes the angle of the mirror at a given time (see Fig. 2).

During the rotation of the mirror, several forces are involved:

- The electrostatic moment $M_e(\theta, V)$, whose expression depends on the configuration of the system, is supposed

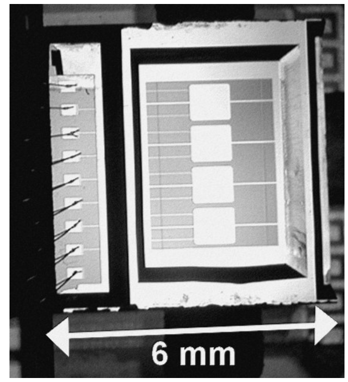


Fig. 1: View of the physical system

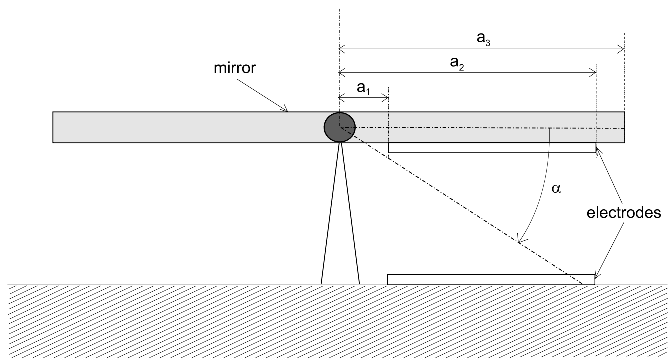


Fig. 2: Cross section of the MEMS with flat electrodes

to be of the form:

$$M_e(\theta, V) = V^2 k(\theta) \tag{10}$$

with V the potential difference between the two electrodes and $k(\theta)$ a positive function with singularity at $\theta = \alpha$.

- The spring moment $M_s(\theta)$; in the standard linear case, $M_s(\theta) = -K\,\theta$, with stiffness constant $K > 0$.
- Friction moment $M_f(\theta, \dot\theta)$; in the standard linear case, $M_f(\theta, \dot\theta) = -\lambda\,\dot\theta$, with $\lambda > 0$.

A dynamic model of such a system is obtained by application of the fundamental principle of dynamics:

$$I\,\ddot\theta - M_f(\theta, \dot\theta) - M_s(\theta) = V^2 k(\theta), \tag{11}$$

$$\text{with the constraint} \quad : \quad |\theta| \leqslant |\alpha|\,, \tag{12}$$

where I is the moment of inertia.

The system (11) of input V and output θ, is completed by the initial conditions: $\theta(0) = \theta_0$ and $\dot\theta(0) = \theta_1$.

For predictive control purposes, such as in [14], [10], [13], [7], [2], we need a reliable model of the system, with good predictive properties. Due to the very small size of these systems, many parameter values cannot be directly measured and dynamic underlying phenomena are difficult to describe from the only physical analysis. In this case, identification process can be the only way to get reliable models. Such an

[3]The picture is published with courtesy of Tronics Microsystems (France).

identification has been performed from measurement data in [6]. The following identified parameters have been obtained:

$$I = 2.693 \times 10^{-16}\,\mathrm{N\,m\,s^2/\,rad};$$

$$M_f(\theta,\dot{\theta}) = -\left(\mu + v\,\theta^3\right)\,\dot{\theta}$$

with $\mu = 1.385 \times 10^{-11}\,\mathrm{N\,m\,s/\,rad}$ and $v = -7 \times 10^{-6}$;

$$M_s(\theta) = -K\theta \quad \text{with} \quad K = 5.366 \times 10^{-8}\,\mathrm{N\,m/\,rad};$$

$$k(\theta) = \sum_{k=0}^{5} c_k\,\theta^{k-1}$$

with $c = (-2.89 \times 10^{-13}, 1.41 \times 10^{-11}, -3.09 \times 10^{-9},$
$-1.5 \times 10^{-7}, -4.96 \times 10^{-6}, 5.76 \times 10^{-4})$.

This identified model has been tested in predictive situation, with voltage values 10 times greater than the ones used for identification [6]. Such predictive results are shown in Fig. 1: the closeness between simulated and experimental trajectories highlights the accuracy of the identified model and so that this model can be used for predictive controls.

Remark 3: (Pull-in phenomenon) When V is a step, by considering the electrostatic and spring moments given in Fig. 4, equilibrium states are obtained when the electrostatic and the spring moments are balanced. As highlighted on Fig. 4, there exists a "pull-in" voltage V_{pullin} such that:

- if $V < V_{\mathrm{pullin}}$, there are two equilibrium points. It can be shown that the first one is stable and the other is unstable;
- if $V = V_{\mathrm{pullin}}$, there is one unstable equilibrium point;
- if $V > V_{\mathrm{pullin}}$, there isn't any equilibrium point: the system "switches" (θ goes to the saturation value α).

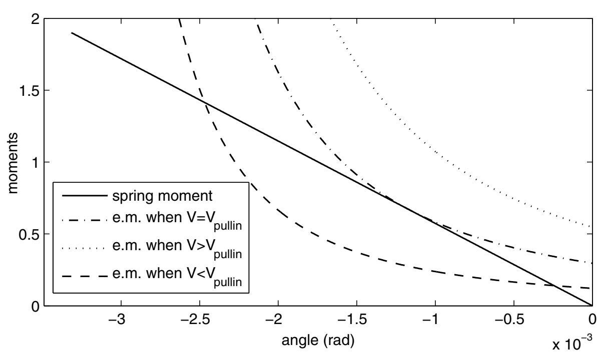


Fig. 4: Spring moment and electrostatic moment (e.m.) for different values of V.

B. Some examples of predictive controls

As long as the constraint is not saturated, model (11) is then equivalent to:

$$\begin{cases} \dfrac{d\theta}{dt} = \dot{\theta} \\ \dfrac{d\dot{\theta}}{dt} = \dfrac{M_f(\theta,\dot{\theta})}{I} - \dfrac{K\,\theta}{I} + \dfrac{V^2 k(\theta)}{I}, \end{cases} \tag{13}$$

and so, (9) becomes in this particular case:

$$V = \sqrt{\frac{1}{k(\theta)}\left(I\,\frac{d^2\theta}{dt^2} - M_f(\theta,\frac{d\theta}{dt}) + K\,\theta\right)}, \tag{14}$$

which allows to exactly compute the input V once the trajectory $y = \theta$ has been chosen (note that this computation can be achieved analytically if θ is chosen analytic; if not, the derivatives of θ must be evaluated numerically by means of precise finite differences for example).

For illustration of the capabilities of the method, three cases are considered for the friction moment:

- the identified visquous friction moment:
  $M_f = -\left(\lambda + \mu\,\theta^3\right)\dot{\theta}$, with values λ, μ defined above,
- visquous friction and dry friction:
  $M_f = -\left(\lambda + \mu\,\theta^3\right)\dot{\theta} - \lambda\,\mathrm{sign}(\dot{\theta})$, with $\lambda = 1.39 \times 10^{-10}\,\mathrm{N\,m}$,
- visquous friction and "hysteretic friction"[4]:
  $M_f = -\left(\lambda + \mu\,\theta^3\right)\dot{\theta} - \lambda\mathbf{g}(\frac{1}{\varepsilon}\dot{\theta})$, with $\mathbf{g}$ the "flip-flop" hysteresis operator[5], $\lambda = 1.39 \times 10^{-10}\,\mathrm{N\,m}$, $\varepsilon = 10^{-6}$.

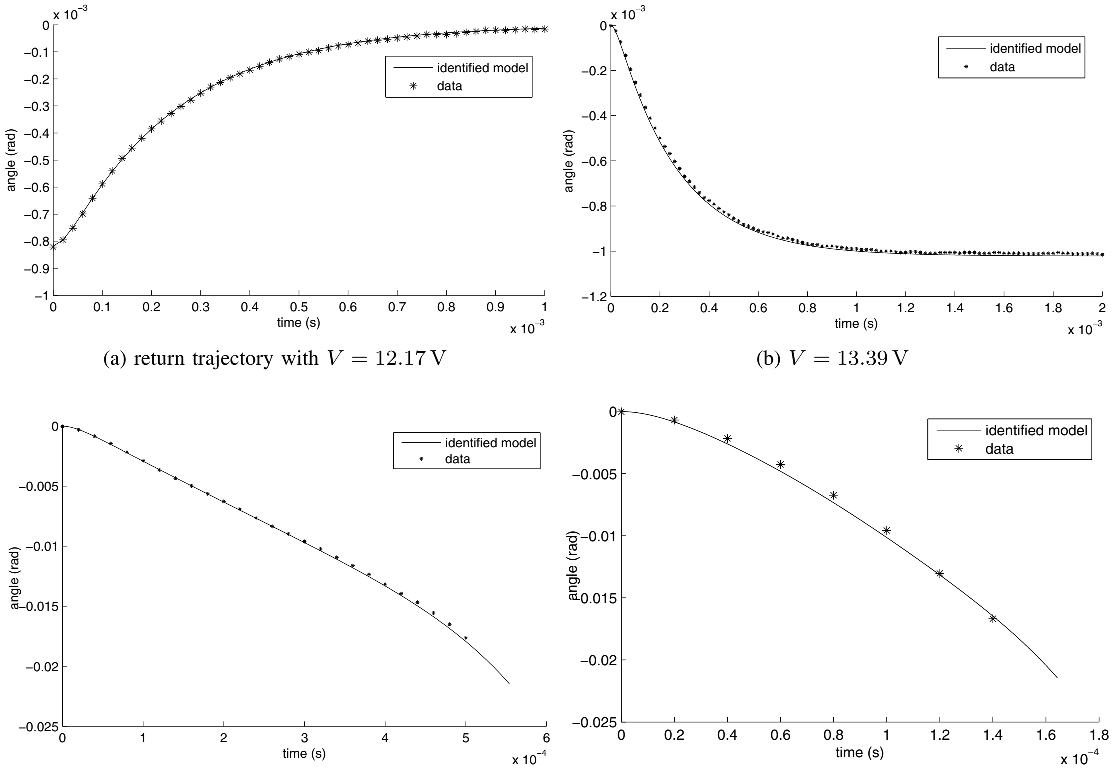
Some examples of controls V are shown in Fig. 5 and Fig. 6. In Fig. 5, we can first compare the V and θ trajectories when the input V is a simple step (note that in that case the trajectory θ has been computed from numerical resolution of (11)) and a "parametrized" control deduced from (14) with a trajectory θ chosen a priori. In this last case, no numerical resolution of (11) is needed, which is a great advantage, from both points of view of precision and numerical cost. Another significant advantage is that the trajectory θ can be chosen almost freely in order to satisfy some possible particular features specific to the problem under consideration. Note that this method allow to choose trajectories that are smooth at the start and at the stabilization time, and then less using for the physical system than "step-input" trajectories.

In Fig. 6, we can see the trajectories associated with parametrized controls in the three cases of friction moment mentioned above. The trajectory θ has been chosen with an overshoot in order to highlight the behavior of the control V when the friction moment switches from a value to the other. For dry as well as "hysteretic" frictions, the control is no more regular but presents discontinuities, when $\dot{\theta}$ vanishes in the case of dry friction, when $\dot{\theta} = \pm\varepsilon$ in the hysteretic case. From a qualitative point of view, such discontinuities were of course predictable; we can however note that the numerical determination of such *singular* controls via the resolution of (11) would be very expensive, in particular for optimal controls which needs numerous evaluations of the trajectories V and θ. On the contrary, the numerical cost when computing a parametrized control is quasi null, thanks to the *explicite and exact* expression (14).

In addition, the trajectories of Fig. 6 are such that the final position of θ is in the unstable zone (that means: $V(t_{\max}) > V_{\text{pull-in}}$). Although unstability does not affect at

[4]This last case is quite academic!

[5]That is: $\mathbf{g}(x > 1) = 1$, $\mathbf{g}(x < 1) = -1$, $\mathbf{g}(x \in [-1, 1]) \in \{-1, 1\}$.



(a) return trajectory with $V = 12.17\,\mathrm{V}$

(b) $V = 13.39\,\mathrm{V}$

(c) $V = 42.60\,\mathrm{V}$

(d) $V = 97.36\,\mathrm{V}$.

Fig. 3: Measured trajectories and the associated θ predicted by the identified model for different values of V

all the computation of the parametrized control, in a physical situation, it would be necessary to stabilize the system by means of a suitable feedback control, as described here-after.

C. On feedback stabilization

Once such a control V has been computed to satisfy a chosen trajectory denoted θ^*, its implementation in open-loop is quite simple. However, some feedback is in practice necessary because first, models are never exact and second, various perturbations are present, with consequence that the real trajectories deviate from the theoretical ones. Such a stabilizing feedback can be judiciously elaborated (for example from classical Lyapunov techniques) on the new model including the previously defined open-loop control, namely:

$$\begin{cases} \dfrac{d\theta}{dt} = \dot{\theta} \\ \dfrac{d\dot{\theta}}{dt} = \dfrac{M_f(\theta,\dot{\theta})}{I} - \dfrac{K\,\theta}{I} + \dfrac{(V+v)^2 k(\theta)}{I}, \end{cases} \tag{15}$$

with $v = v(\theta - \theta^*)$ the feedback control to be designed. By assuming that v remains small, and so that $(V+v)^2 \simeq V^2 + 2V\,v$, the following new model can then be considered:

$$\frac{d\dot{\theta}}{dt} = \frac{M_f(\theta,\dot{\theta})}{I} - \frac{K\,\theta}{I} + \frac{V^2 k(\theta)}{I} + \frac{2V\,k(\theta)}{I} v;$$

by denoting:

$$f(V,\theta) = \frac{2V\,k(\theta)}{I} > 0,$$

and by defining a new time τ as defined in section II:

$$\tau = \int_0^t f(V(s),\theta(s))\,ds,$$

model (15) rewrites[6]:

$$\begin{cases} \dfrac{d\tilde{\theta}}{d\tau} = \dfrac{I\,\widetilde{\dot{\theta}}}{2\,\tilde{V}\,k(\tilde{\theta})} \\ \dfrac{d\widetilde{\dot{\theta}}}{d\tau} = \dfrac{M_f(\tilde{\theta},\widetilde{\dot{\theta}})}{2\,\tilde{V}\,k(\tilde{\theta})} - \dfrac{K\,\tilde{\theta}}{2\,\tilde{V}\,k(\tilde{\theta})} + \dfrac{\tilde{V}}{2} + \tilde{v}; \end{cases} \tag{16}$$

the stabilizing feedback control v is now simply additive, and so much more simple to design.

V. Conclusion

The simple numerical results presented above have highlighted the efficiency of the trajectorial approach for predictive control problems. This approach is mainly based on the explicite knowledge of a "parametric output". When it exists, such an output is not always simple to find. For a large class of MEMS models however, it is simply a part of the state, as previously mentionned.

From the feedback stabilization point of view, further trajectorial features can be involved in order to improve the performance of the control. For example, sophisticated feedbacks

[6]With $\tilde{x}(\tau) := x(t(\tau))$ for any function x.

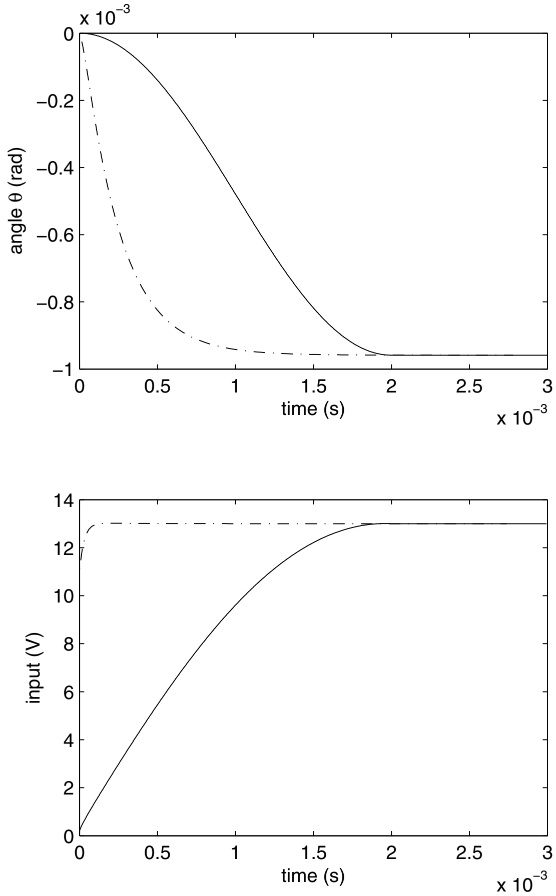


Fig. 5: Trajectories with predictive control (-) and for a step V (- -)

elaborated from *trajectorial linearization* of the model (16) should allow better compromises between gain and robustness than static linearizations. This is currently under study.

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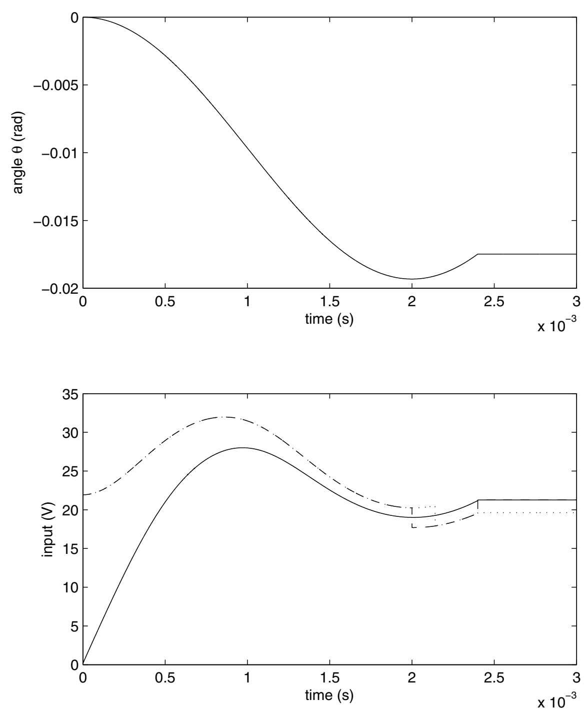

Fig. 6: Trajectories with predictive control: with identified friction moment (-), with dry friction (-.-) and with "histeretic" friction (..)

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